Analysis of the Acoustic Planform Method for Rotor Noise Prediction

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Abstract

THIS study analyzes the acoustic planform method as an alternative to using the equation of Ffowcs Williams and Hawkings for predicting transonic and supersonic rotor noise. The studied method avoids the singularity encountered when the noise source travels towards the observer at sonic velocity. It introduces the necessity for computing acoustic planforms and integrating over them. Results are presented for a rotating, rectangular, monopole surface with supersonic tip velocity. These computations show a decrease in peak acoustic pressure as the tip speed increases beyond a critical supersonic Mach number. The results provide an explanation for some experimental data and some guidelines for proceeding on to the prediction of actual rotor noise.

Contents

The acoustic planform method, though utilized previously,[1-3] has not been extensively analyzed in the literature. Farassat[1] gives a derivation of the relevant equation which provides an excellent mathematical background for the problem at hand. The integral equation for the acoustic pressure, p', due to a moving surface including nonlinear source terms, is

$$4\pi p' = \frac{\partial^2}{\partial x_i \partial x_j}\int\int\int_{-\infty}^{\infty}\left[\frac{T_{ij}}{r\Lambda}\right]\mathrm{d}\bar{y} - \frac{\partial}{\partial x_i}\int\int\left[\frac{p_{ij}n_j}{r\Lambda}\right]\mathrm{d}\Sigma + \frac{\partial}{\partial t}\int\int\left[\frac{\rho_0 v_n}{r\Lambda}\right]\mathrm{d}\Sigma \qquad (1)$$

T_{ij} is the Lighthill stress tensor, a term which accounts for noise sources not directly caused by the moving surface. The square brackets indicate evaluation of the integrand at the retarded time, and integration takes place over a volume or surface at the retarded time—an acoustic planform. The value of $\Lambda = (1 + M_n^2 - 2M_n\cos\theta)^{1/2}$, where M_n is the Mach number normal to the surface and θ the angle between M_n and the observer direction, is close to 1 for thin airfoil shapes except in the vicinity of a blunt leading edge.

Those points from which all sound reaching the observer at a given time emanates make up an acoustic planform. The planform for a rotating surface source, such as a hovering rotor, with observer in the plane of rotation, may be computed by solving for all values of ψ from the equation for dimensionless observer time:

$$\tilde{t} = \psi + \delta + \tilde{r} \qquad (2)$$

In the above, ψ = angular location of a point on the acoustic planform, δ = angular offset, and $\tilde{r} = \omega r/a_0$, the dimensionless distance from the point on the planform to the observer. ω is the angular velocity of the rotating surface and $\tilde{t} = \omega t$.

For a known R (distance to the observer), the shape of the function given by Eq. (2) and the number of (ψ) roots at a given observer time depend on the local Mach number, M. A hovering surface will experience Mach numbers from 0 to M_{tip}, but supersonic regions provide the most interesting study. The relationship between ψ and $\tilde{t}$ for $R = 4.5$ and $M = 1.4$ is shown in Fig. 1a for two different δ's. Multiple roots occur when the component of the source Mach number in the direction of the observer, M_r, is greater than 1. This implies that there are multiple emission times for a single observer time.

The third term on the right hand side of Eq. (2) contains the monopole source which contributes the "thickness" noise of a hovering rotor. In dimensionless terms, the equation containing only monopole sources becomes

$$2\pi C_p = \frac{\partial}{\partial \tilde{t}}\int\int\left[\frac{M_n}{\tilde{r}\Lambda}\right]M\,\mathrm{d}\psi\,\mathrm{d}M \qquad (3)$$

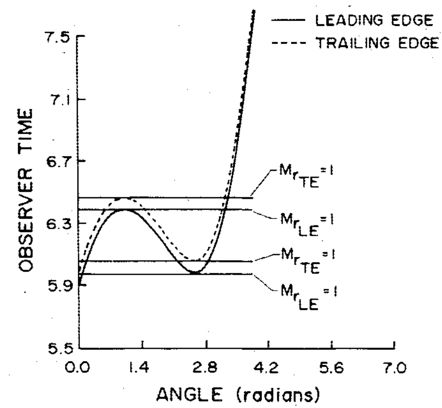

Fig. 1a Relationship between observer time and emission angle, $M = 1.4$.

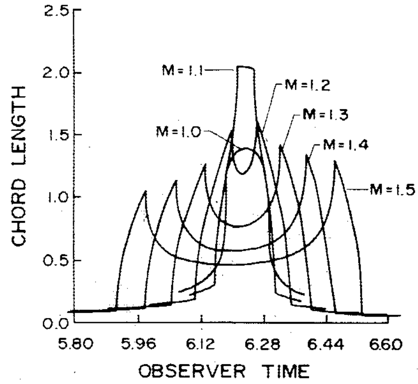

Fig. 1b Acoustic planform chord length vs observer time.

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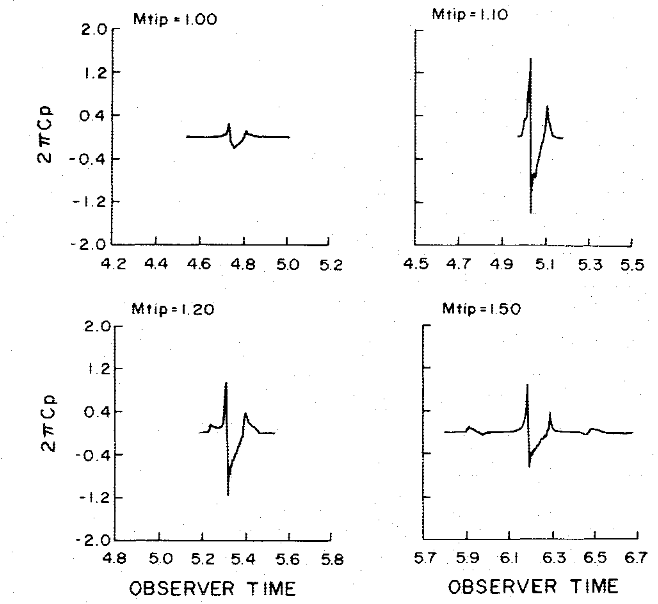

Fig. 2 Pressure signatures—transonic and supersonic tips (monopole sources).

where $M\,d\psi\,dM$ is a dimensionless differential surface element, or $d\Sigma$, in cylindrical coordinates. The integration limits on the angular variable can be computed to any desired accuracy by solving Eq. (2) for ψ using the δ corresponding to leading and trailing edges of the rotating surface. Neglecting for the time being the special case of $\Lambda = 0$, the integrand of Eq. (3) contains no singularities when evaluated in the far field. The interest (and difficulty) of the integration and subsequent differentiation lies in the variation of the acoustic planform area with time.

The acoustic planform area may be computed at any observer time from

$$A = \int\int M\,d\psi\,dM \tag{4}$$

The inner integral, $\int M\,d\psi$, gives the acoustic planform chord length at a given M where the limits on ψ are computed from Eq. (2). Figure 1b illustrates the behavior of planform chord length as a function of observer time at several supersonic radial locations. The discontinuities in slope occur at times corresponding to $M_r = 1$ at the leading or trailing edge of the source surface. As shown by Fig. 1a for $M = 1.4$, a sudden change occurs in the size of the acoustic planform at these times as evidenced by a change in the number of roots of the function describing it. Note that the chordwise length of the acoustic planform is determined by the horizontal distance between the curves in Fig. 1a.

Since the radial or spanwise integration consists of essentially carrying out a weighted sum of the contributions from all curves at one $\tilde{t}$ in Fig. 1b, it becomes apparent that a peak must be included at any observer time at which the integral is computed. Failure to include the appropriate radial location in the integration grid will result in an underestimate of the acoustic planform area.

Upon integration, the interior peaks are smoothed. The initial and final slope discontinuities in acoustic planform area (those for $M = 1.5$ in the example figure), however, remain. Because the monopole acoustic pressure depends on the time derivative of an area integral, a discontinuity should appear in the acoustic signature regardless of the source strength in the integrand. Numerical differentiation tends to smear this effect, but the peak pressure does not occur at the times of discontinuous slope.

Figure 2 presents some results of computing acoustic signatures using the acoustic planform method. All data is for a rectangular rotating surface of aspect ratio 14 and $R = 3M_{tip}$.

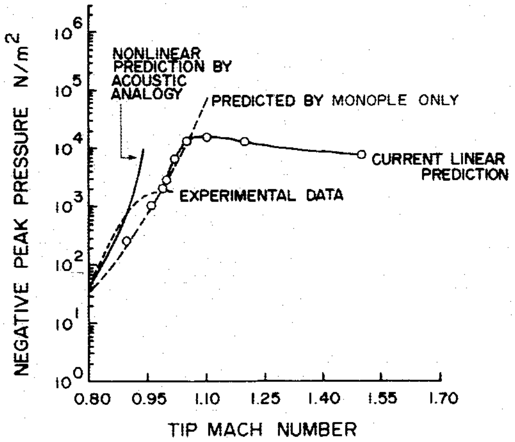

Fig. 3 Negative peak pressure vs tip Mach number—comparison with previous results.

The cross-sectional shape is essentially that of a NACA 0012 airfoil but with a sharp leading edge to avoid the singularity caused when $\Lambda = 0$. Though the signal width increases as tip Mach number increases, the peak pressures decrease above some tip Mach number around 1.1. The decrease occurs because of acoustical interference between the various regions of the blade traveling at different Mach numbers. The derivative of the planform area goes from positive to negative at different times for different Mach numbers. When added together, then, a cancellation between positive and negative derivatives occurs at some observer times.

A glance at the Ffowcs Williams-Hawkings[4] equation makes the above results initially seem obvious. That equation contains the quantity $|1 - M_r|$ in the denominator of the source integrands, implying an infinity and, therefore, a maximum at $M_r = 1$. However, a rotating line of sources with supersonic tip will have a continuous line of locations where $M_r = 1$, including one point where $M_r = 1$ and $M = 1$. Thus, M_r equals 1 at many locations on a rotating surface with a supersonic tip, so that the decrease in peak pressure with increasing Mach number must be due to acoustical interference rather than to the Doppler factor.

Recent results from actual supersonic rotor noise calculations (incorporating the nonlinear source terms)[2,5-6] unanimously overpredict the measured acoustic pressure. The example computations from Ref. 2 are shown in Fig. 3. Equation (1) is an exact result as long as the integrals contain all acoustic sources. It is hypothesized, based on the current results for supersonic tip speeds, that these studies do not include enough of the source region in the integration domain to encounter the acoustic interference effect. Results from these monopole surface calculations indicate that extension of the source region into the supersonic flow will produce the bend-over in the peak pressure curve (Fig. 3) which is evident in the experimental results.

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